A Bio-Inspired Framework for Joint Angle Estimation from Non-Collocated Sensors in Tendon-driven Systems

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Abstract*— Estimates of limb posture are critical for the control of robotic systems. This is generally accomplished by utilizing on-location joint angle encoders which may complicate the design, increase limb inertia, and add noise to the system. Conversely, some innovative or smaller robotic morphologies can benefit from non-collocated sensors when encoder size becomes prohibitively larger or the joints are less accessible or subject to damage (e.g., distal joints of a robotic hand or foot sensors subject to repeated impact). These concerns are especially important for tendon-driven systems where motors (and their sensors) are not placed at the joints. Here we create a framework for joint angle estimation by which artificial neural networks (ANNs) use limited-experience from motor babbling to predict joint angles. We draw inspiration from Nature where (i) muscles and tendons have mechanoreceptors, (ii) there are no dedicated joint-angle sensors, and (iii) dedicated neural networks perform sensory fusion. We simulated an inverted pendulum driven by an agonist-antagonist pair of motors that pull on tendons with nonlinear elasticity. We then compared the contributions of different sets of non-collocated sensory information when training ANNs to predict joint angle. By comparing performance across different movement tasks we were able to determine how well each ANN (trained on the different sensory sets of babbling data) generalizes to tasks it has not been exposed to (sinusoidal and point-to-point). Lastly, we evaluated performance as a function of amount of babbling data. We find that training an ANN with actuator states (i.e., motor positions/velocities/accelerations) *as well as* tendon tension data produces more accurate estimates of joint angles than those ANNs trained without tendon tension data. Moreover, we show that ANNs trained on motor positions/velocities *and* tendon tensions (i.e., the *bio-inspired* set) (i) can reliably estimate joint angles with as little as 2 minutes of motor babbling and (ii) generalizes well across tasks. We demonstrate a novel framework that can utilize limited-experience to provide accurate and efficient joint angle estimation during dynamical tasks using non-collocated actuator *and* tendon tension measurements. This enables novel designs of versatile and data-efficient robots that do not require on-location joint angle sensors.

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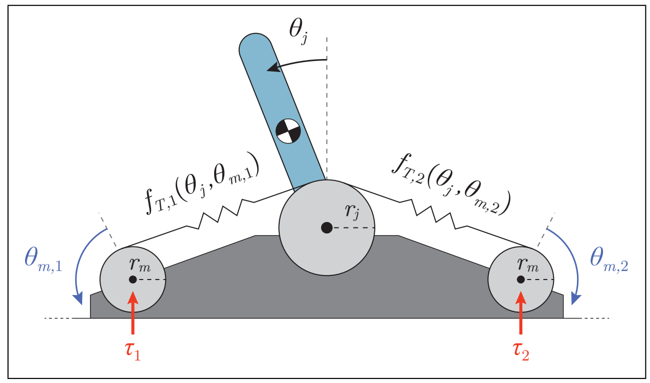

Fig. 1. Schematic of tendon-driven system with 1 kinematic degree of freedom and 2 degrees of actuation (motors) that pull on tendons with nonlinear elasticity (creating a tension, $f_{T,i}$). The motors are assumed to be backdriveable with torques (τ_i) as inputs.

I. INTRODUCTION

Tendon-driven robots are becoming popular due to a number of advantages these designs can provide [1]–[3]. Elastic tendons can increase energy efficiency by storing potential energy and can protect actuators from impacts by dissipating energy upon impact [4]–[7]. Additionally, tendon routings offer flexibility to how torques and angular velocities at the motors are converted to torques and angular velocities at the joints [8]–[11]. Most importantly, tendon-driven systems offer flexible placement options for the actuators, which eliminate the need for motors to be placed on the joints themselves. Proximal actuator placement moves the center of mass towards the body of the robot thereby reducing limb inertia and allowing for more efficient displacement in quadrupeds or anthropomorphic robots [12].

However, most successful state-based robotic control strategies need to observe or approximate joint angles which is generally done by placing sensors on the joints [3], [13] (in the absence of alternatives such as visual feedback). Although sensors in general have lighter mass than motors, this can still add unwanted inertia to the limbs. These on-location sensors are prone to motion noise and their wiring is often cumbersome and poses a potential risk of damage. These adverse effects become more pronounced for smaller, distal joints where the mechanical design may make the joint inaccessible (e.g., in the case of a tendon-driven finger in a robotic hand). One alternative solution, which biology seems to take advantage of, is to have non-collocated sensors (i.e., in the muscle and tendon instead of the joint) and use fusion of sensory information from actuators and tendons to predict joint angles. It is interesting to note that biological systems do not seem to have dedicated sensors that explicitly and uniquely encode joint angles. Instead, they have sensors for muscle (actuator) lengths and velocities (called *muscle spindles*, [14]) and for tendon tensions (called *Golgi tendon organs*, [15])[1]. Previous work has emphasized that a functional (yet indirect) relationship exists between sensory states in general and kinematic states (like posture) [1], [17], [18]. It is therefore speculated that these sensory signals may be integrated through their spinal and supraspinal projections to form internal representations of expected or virtual limb position [19]–[22]. The existence (and possible use) of this indirect relationship between sensory states and kinematic states in biology implies it may be possible to use sensory fusion in tendon-driven robots to infer joint angles from actuator (e.g., motor angles) and structural (e.g., tendon tensions) sensors, thereby removing the need for on-location joint angle encoders.

While it is sometimes possible to derive analytical relationships among tendon tensions, motor rotations, and joint posture given the precise equations for the kinematics and dynamics, in practice it is often impractical or impossible to obtain accurate and time-invariant models of such nonlinear dynamical systems [3], [23]. Furthermore, even if an accurate model of the system were available, these relationships (i) would not generalize across changes in mechanical designs or tasks and (ii) will become increasingly inaccurate as the plant suffers mechanical changes due to either damage or normal wear and tear [24]. Therefore, data-driven systems that can efficiently create mappings between sensory information are preferred in practical applications [23], [25], [26].

Here we introduce a framework to train artificial neural networks (ANNs) from limited-experience via *motor babbling* to be able to predict joint angles from sets of non-collocated sensory information. As a proof of concept, we simulated an inverted pendulum (controlled by two motors that pull on tendons with nonlinear elasticity), trained ANNs on different sets of sensory information (i.e., actuator and/or tendon tension data) for different durations of motor babbling, and evaluated the performance of the ANNs and their ability to accurately predict joint angle for different unlearned movement tasks.

II. METHODS

A. Plant Definition and Equations

In order to determine the utility of observing different sets of sensory information in a tendon-driven system it is important to have a consistent model across trials—such as a standard numerical simulation. As we aimed to conduct a thorough and systematic experiment, it was also impractical to use a physical system that is prone to imperfect modelling and time-varying changes to physical parameters. For those reasons, we have simulated a simple 1 degree of freedom tendon-driven system with 2 degrees of actuation that pull on tendons with nonlinear stiffness (Fig. 1) such that we can know/control the parameters that govern the system's dynamics and we can reliably conduct many experiments on the same plant. We modelled the actuators as brushed DC motors with no gearing to allow for the motors to be backdriveable and considered the input to be motor torques (τ_i). Similar to [27], the tension on the tendons ($f_{T,i}$, (1)) depends nonlinearly (exponentially) on tendon deformation ($\varepsilon_{T,i}$), which is a function of the pendulum joint angle (θ_j) and the motor angle ($\theta_{m,i}$).

$$f_{T,i}(\theta_{m,i},\theta_j) = k_T\left(\exp(b_T\varepsilon_{T,i})-1\right) \tag{1}$$

$$\text{where } \varepsilon_{T,i} = \begin{cases} r_m\theta_{m,1} - r_j\theta_j; & i=1 \\ r_m\theta_{m,2} + r_j\theta_j; & i=2 \end{cases}$$

and $b_T > 0$, $k_T > 0$ are shape constants.

The equations of motion for this system can therefore be written as (2) where G is the torque due to gravity, and I, D, and r represent the moment of inertia, damping coefficient, and moment arm values of either the joint or the motors (denoted by the subscripts j and m, respectively, and chosen from [27]).

$$\begin{cases} \ddot{\theta}_j = \dfrac{1}{I_j}\left[-D_j\dot{\theta}_j - G(\theta_j) + r_j\left(f_{T,1}(\theta_j,\theta_{m,1}) - f_{T,2}(\theta_j,\theta_{m,2})\right)\right] & \text{(2a)} \\ \ddot{\theta}_{m,i} = \dfrac{1}{I_{m,i}}\left[-D_m\dot{\theta}_{m,i} - r_m f_{T,i}(\theta_j,\theta_{m,i}) + \tau_i\right] \quad (\text{for } i\in\{1,2\}) & \text{(2b)} \end{cases}$$

Lastly, we express this system of equations by its state space representations, (3), where the $\vec{x} = [\theta_j, \dot{\theta}_j, \theta_{m,1}, \dot{\theta}_{m,1}, \theta_{m,2}, \dot{\theta}_{m,2}]^T$, $\vec{u} = [\tau_1, \tau_2]^T$, and $y = h(\vec{x}) = \theta_j$.

$$\begin{cases} \dot{\vec{x}} = f(\vec{x}) + g(\vec{x})\vec{u} & \text{(3a)} \\ y = h(\vec{x}) & \text{(3b)} \end{cases}$$

B. Motor Babbling Experiments

In order to efficiently learn a mapping from a particular sensory set ($\vec{x}^{i}_{\text{sens}}$) to joint angles, we performed motor babbling experiments whereby we (i) passed a series of random input torques to the motors, (ii) recorded all subsequent sensory information, and then (iii) trained an ANN to predict joint angle from that specific sensory set (Fig. 2). The babbling input torques were generated from low frequency, band-limited white noise (1-10 Hz), offset by uniformly selected input values. The offset value changed every 4 seconds on average (0.25 Hz) and the amplitude of the noise was chosen to restrict the input values within the allowable range. This particular type of babbling differs from the step inputs used in [3] in that here we have added low

[1] There are additional biological sensors that detect stretch in the skin and synovial capsule, but these do not *directly* encode joint position either [16].

frequency white noise to the step inputs which allows us to search the solution space around equilibrium postures in the hope of sampling the nullspace.

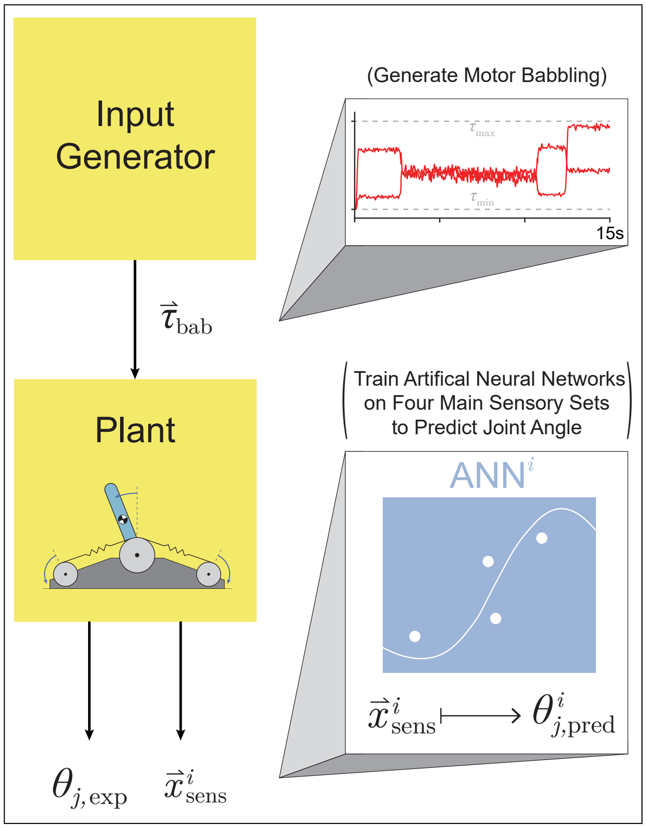

Fig. 2. Proposed setup for training artificial neural networks on motor babbling. Random input torques were generated from low frequency, band-limited white noise (1-10 Hz), offset by uniformly selected input values that change every 4 seconds on average. These motor babbling signals are passed through the plant and all subsequent sensory information is recorded. Lastly, an artificial neural network is trained on a particular set of sensory information ($\vec{x}^{\,i}_{\text{sens}}$) to predict joint angle ($\theta^i_{j,\text{pred}}$).

C. Training & Testing Artificial Neural Networks

In order to observe the effect (if any) that the duration of motor babbling has on an ANN's ability to predict joint angle from a specific sensory information set, the subsequent experiments were conducted for babbling durations between 30 and 360 seconds (*resolution*: 15 seconds). Four separate sensory information sets were considered throughout this paper: First, as a baseline set, we consider the set of all motor states and tendon tensions as in (4).

$$\vec{x}^{\,1}_{\text{sens}} = \begin{bmatrix} \vec{\theta}_m^{\,T} & \dot{\vec{\theta}}_m^{\,T} & \ddot{\vec{\theta}}_m^{\,T} & \vec{f}_T^{\,T} & \dot{\vec{f}}_T^{\,T} & \ddot{\vec{f}}_T^{\,T} \end{bmatrix}^T \in \mathbb{R}^{12} \tag{4}$$

We then consider the *bio-inspired* set, which observes motor position and velocities and tendon tensions (5). This set is reminiscent of the sensory signals available to biological systems from the non-collocated muscle spindles and Golgi tendon organs, respectively.

$$\vec{x}^{\,2}_{\text{sens}} = \begin{bmatrix} \vec{\theta}_m^{\,T} & \dot{\vec{\theta}}_m^{\,T} & \vec{f}_T^{\,T} \end{bmatrix}^T \in \mathbb{R}^{6} \tag{5}$$

The last two sets, given by (6) & (7), consider the sensory sets that do not include tendon tensions in any form. The first considers motor position and velocities *only* (this set parallels a hypothetical biological system that only uses spindle information to determine joint posture), and the second incorporates motor acceleration to see if it can provide useful information about the plant dynamics not captured by kinematics alone.

$$\vec{x}^{\,3}_{\text{sens}} = \begin{bmatrix} \vec{\theta}_m^{\,T} & \dot{\vec{\theta}}_m^{\,T} \end{bmatrix}^T \in \mathbb{R}^{4} \tag{6}$$

$$\vec{x}^{\,4}_{\text{sens}} = \begin{bmatrix} \vec{\theta}_m^{\,T} & \dot{\vec{\theta}}_m^{\,T} & \ddot{\vec{\theta}}_m^{\,T} \end{bmatrix}^T \in \mathbb{R}^{6} \tag{7}$$

For each babbling duration, 50 different babbling experiments were conducted. For each experiment, a ANN (with one hidden layer of 15 nodes) was constructed for each of the four sensory sets where it was trained on 90% of the babbling data and then tested on the remaining 10% for 50 epochs (divided randomly). Previous work determined this architecture to be sufficient in the case of rigid tendons and sufficed as an initial guess for this experiment. The performance for each epoch was given by the RMS error with the performance of the final epoch serving as the *testing* performance of each ANN. The traces of testing performance on babbling data versus epoch number were averaged over the 50 trials to produce more robust curves where trends can be more easily identified. When we overlap the average testing performance versus epoch number curves for each babbling duration, we get Fig. 3. From these plots we can identify two main trends; (i) when testing on a subset of

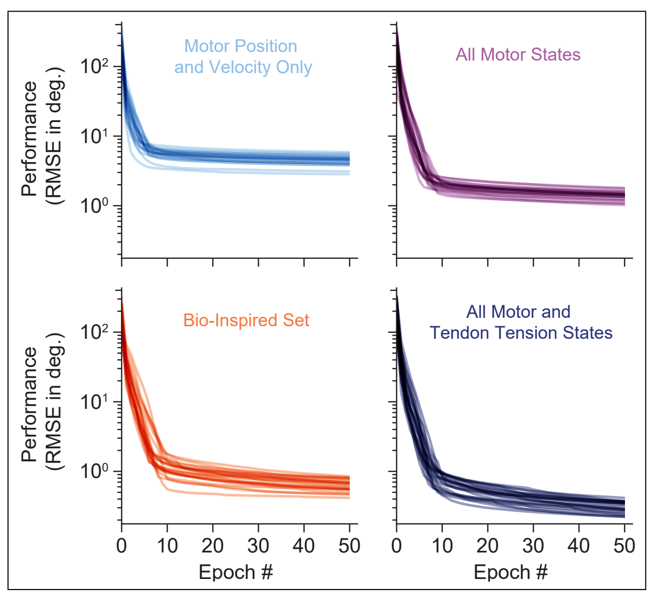

Fig. 3. Overlaid plots of testing performance on babbling data (RMS error in degrees) versus epoch number for all babbling durations of interest (30,45,...,360 seconds). For the four sensory sets of interest, the overall trend of each plot is conserved across changes in babbling duration. Additionally, the artificial neural network trained on the *bio-inspired* set (i.e., motor positions/velocities and tendon tensions) performed better than those neural networks trained without tendon tension data.

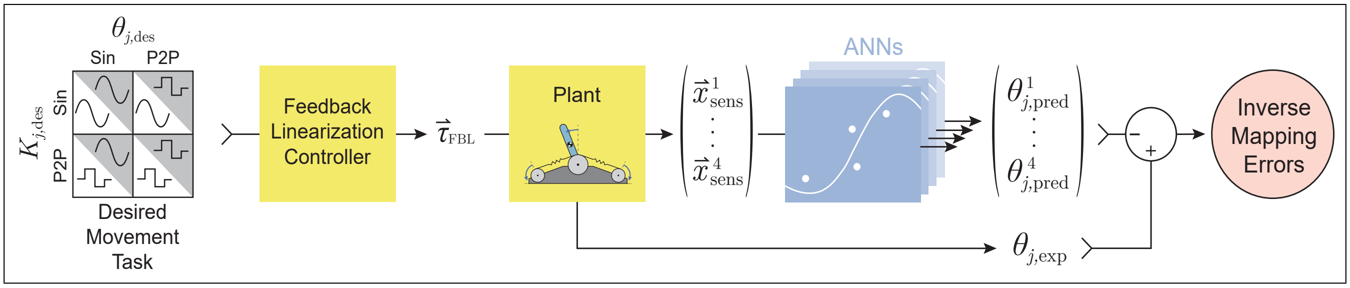

Fig. 4. Proposed flow chart for testing artificial neural network generalizability. First, one of the four movement tasks of interest is chosen (where joint angle and/or stiffness are prescribed either sinusoidal (Sin) or point-to-point (P2P) trajectories). A feedback linearization controller then calculates the input torques needed to produce the desired movements, which are then passed through the plant to produce the experimental joint angle ($\theta_{j,\mathrm{exp}}$) as well as the four sensory sets of interest ($\vec{x}^{i}_{\mathrm{sens}}$). These sets are then passed through the artificial neural networks (ANNs) that were trained on babbling data to predict joint angle ($\theta^{i}_{j,\mathrm{pred}}$). The experimental and predicted value can then be compared to determine each ANN's ability to generalize to each movement task.

the babbling data, networks trained with tendon tension data (Fig. 3 *bottom*) outperform those networks trained without them (Fig. 3 *top*), and (ii) this trend (as well as the overall shape of each curve) do not seem to depend on babbling duration. This second observation is to be expected as the networks will only be tested on data similar to the training data. Therefore, in order to address the effect that babbling duration has on the overall performance of these predictive ANNs, we must consider how they ANNs *generalize* to different types of movement tasks that the ANNs have not been exposed to.

D. Generalizing to Different Movement Tasks

While it is important to understand how these ANNs performed on the test data (random 10% sample of babbling data) to identify whether different sensory sets *can* be used to predict joint angles sufficiently, it is perhaps more important to understand if these limited-experience ANNs can *generalize* to different types of movements. To adequately address each ANNs ability to generalize, we needed to identify movement tasks that were representative of most typical movements. An enabling feature of tendon-driven system with tendons with nonlinear stiffness is the ability to control joint angle independently of joint stiffness (K_j, (8)) by choosing different tendon tensions in the nullspace of the joint dynamics. Therefore, by defining four different movement tasks where joint angle and stiffness were prescribed either sinusoidal or random point-to-point trajectories within the range of these values (10 min @ 1 kHz), we were able to generate movement tasks that categorized most typical movements. The sinusoidal trajectories were assigned random frequencies between 0.125 and 1 Hz to ensure that the movements were not (i) too fast or (ii) phase locked. The amplitude was chosen to sample the entire range of either joint angles or joint stiffness. The point-to-point trajectories were generated by randomly jumping to values that were uniformly sampled from the range of possible values every 4 seconds on average (0.25 Hz) [3].

$$K_j = \frac{\partial}{\partial \theta_j}\left[\frac{r_j}{I_j}\big(f_{T,1}(\theta_{m,1},\theta_j) - f_{T,2}(\theta_{m,2},\theta_j)\big)\right] \qquad (8)$$

In order to calculate the sensory sets associated with each movement task, a feedback linearization algorithm was used. The reader is directed to [27] for a more thorough explanation of the controller, but in short, if we consider the outputs of the system to be $y = h(\vec{x}) = [\theta_j, K_j]^T$, then we can completely transform the nonlinear system of equations, (3), into a linear system that can be controlled by feedback. We find that we must differentiate the joint angle 4 times and the joint stiffness 2 times in order for the input torques to appear, which corresponds to a total relative degree of 6 for a system of equations with 6 states, which is a criterion for use of feedback linearization, (9).

$$\begin{cases} y^{[4]}_{\theta_j} = L_f^4 h_{\theta_j}(\vec{x}) + L_g L_f^3 h_{\theta_j}(\vec{x})\vec{u} & \text{(9a)} \\ \ddot{y}_{K_j} = L_f^2 h_{K_j}(\vec{x}) + L_g L_f h_{K_j}(\vec{x})\vec{u} & \text{(9b)} \end{cases}$$

$$\vec{u} = \begin{bmatrix} L_g L_f^3 h_{\theta_j}(\vec{x})\vec{u} \\ L_g L_f h_{K_j}(\vec{x})\vec{u} \end{bmatrix}^{-1} \left(\begin{bmatrix} v_{\theta_j} \\ v_{K_j} \end{bmatrix} - \begin{bmatrix} L_f^4 h_{\theta_j}(\vec{x}) \\ L_f^2 h_{K_j}(\vec{x}) \end{bmatrix} \right) \qquad (10)$$

By choosing $\vec{u}$ to be (10), we can choose $\vec{v} = [v_{\theta_j}, v_{K_j}]^T$ such that the (now linear) system stabilizes around the prescribed reference trajectories. Note that $L_f^n h_{\theta_j}$ and $L_f^n h_{K_j}$ represent the n-th Lie derivatives of the functions $h_{\theta_j}(\vec{x}) = \theta_j$ and $h_{K_j}(\vec{x}) = K_j$ along $f(\vec{x})$, respectively. Additionally, $L_g L_f^3 h_{\theta_j}(\vec{x})$ and $L_g L_f h_{K_j}(\vec{x})$ are the Lie derivatives of $L_f^3 h_{\theta_j}(\vec{x})$ and $L_f h_{K_j}(\vec{x})$ along $g(\vec{x})$, respectively, where $L_g L_f^n h_{\theta_j}(\vec{x}) = 0$ for $i \in \{1,2\}$ and $L_g h_{K_j}(\vec{x}) = 0$. It is important to note that the use of a feedback linearization controller (that relies on an accurate model of the system) was used to (i) prescribe desired movement trajectories and (ii) fully observe the internal sensory states. As such, it is not a part of the proposed framework to build ANNs to predict joint angle but instead a tool used to generate systematically varied movements and their associated sensory sets needed to test the performance and generalizablity of the proposed networks.

Once the experimental sensory sets were generated for each movement type, we tested how well each ANN was able to recover the actual joint angle of the system by calculating the RMS error of the predicted joint angle and the actual joint angle (given by the forward simulation of the plant). This was repeated 50 times (50 Monte Carlo runs) for each babbling duration so that we may calculated the average RMS error across trials. For more explanation of how these ANNs were trained and tested, please see the supplementary video that demonstrates how one set of ANNs generalized to a specific task.

III. RESULTS

As we can see in Fig. 3, ANNs trained with tendon tension data outperform those networks trained without them. We can see that ANNs trained with the *bio-inspired* set will generally converge to a performance value comparable to the ANNs trained with the baseline set ($< 1°$ RMS error), which is better than the performance of the ANNs trained without tendon tension. As previously mentioned, while it is useful to know that incorporating tendon tensions in these ANNs produces better joint angle estimates when testing on babbling data, the results on Fig. 3 do not show how these ANNs *generalize* to movements that are different than the babbling data. Therefore it was necessary to test these ANNs on different types of movements to observe how well they generalize and how the duration of babbling affects the performance.

We can see from Fig. 5, where we plot average RMS error versus babbling duration, that for each movement type the *bio-inspired* set generalizes (i) better than those ANNs trained without tendon tension data and (ii) nearly as well as the baseline (i.e., full) set for babbling durations longer than 2 minutes. It is important to note that the babbling data is completely random and as such we will expect some jaggedness in the results. We can also see that the ANN trained on all motor states generalized well for longer babbling durations ($\geq$ 5 min) for every task except the one with point-to-point joint angle and sinusoidal joint stiffness trajectories (Fig. 5, *top right*). Lastly, the ANN trained on just motor position and velocity data performed poorly across all movement tasks, in agreement with the testing performance on babbling data. From these results, we find that an ANN can be trained with a *bio-inspired* set of sensory data for *as little as 2 minutes of motor babbling* to produce a joint angle predictor that (i) performs comparably to an ANN trained on all sensory data and (ii) generalizes to most types of movements. It is worth noting that the standard deviations for the ANNs trained on the *bio-inspired* set and the set of all motor and tendon tension states are comparable as the performances converge (see supplementary figures on the Github repository).

IV. DISCUSSION

We created a framework for joint angle estimation by which artificial neural networks (ANNs) use limited-experience from motor babbling to predict joint angles. As vertebrate animals seamlessly learned to control redundant tendon-driven limbs with (i) *no dedicated joint-angle sensors* by (ii) combining different non-collocated sensory information (e.g., muscle length via muscle spindles and tendon tension via Golgi tendon organs) we therefore explored

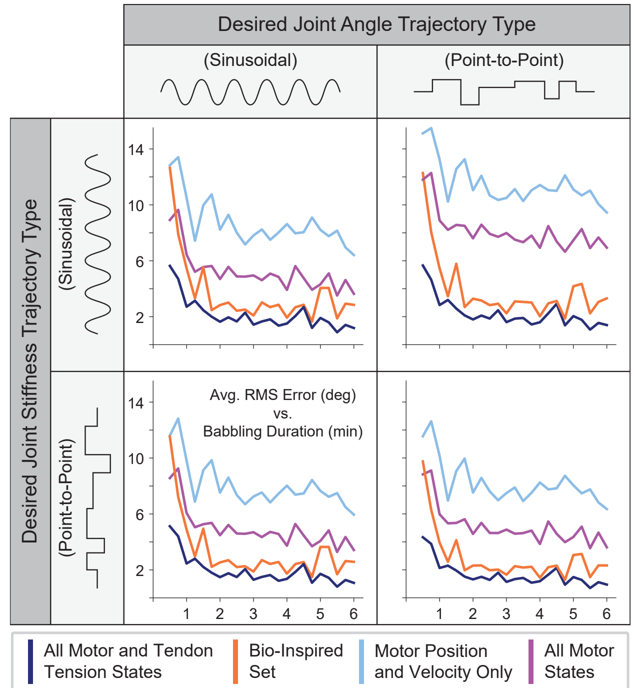

Fig. 5. Plots of the average performance (RMS error) versus babbling duration (minutes). For each babbling duration, 50 artificial neural networks (ANNs) were trained from babbling data and the average error when testing on each of the four movement tasks were plotted. The ANN trained on the *bio-inspired* set (i) outperforms the ANNs trained without tendon tension data and (ii) approaches the performance of the baseline set ("All Motor and Tendon Tension States") for babbling durations greater than 2 minutes. Please see the video online for a demonstration of how these ANN were trained and how they performed for different tasks.

whether and how ANNs might predict joint angles when trained on different sets of sensory data (each containing different information) in a tendon-driven robotic system. We found that ANNs trained with sets of sensory data generated from motor babbling that include tendon tension (either the *bio-inspired* set or the set of all motor and tendon tension states) performed better than those ANNs trained with sets of sensory data that did not include tendon tension data (Fig. 3). Specifically, the *bio-inspired* set (Fig. 3, *bottom left*) performs comparably to the set of all motor and tendon tension states (Fig. 3, *bottom right*) when testing on a subset of babbling data with RMS prediction errors $< 1°$.

By comparing performance across different movement tasks we were able to determine (i) how well each ANN (trained on the different sets of sensory data generated by motor babbling) generalizes to tasks it was not exposed to (sinusoidal and point-to-point) and (ii) what effect (if any) the amount of sensory data (i.e., duration of motor babbling) has on each ANN's performance. In Fig. 5, we found that, in general, the ranking of average ANN performance when generalizing to different dynamical tasks was consistent with the training performance ranking seen in Fig. 3. Additionally, we found that ANNs trained on the *bio-inspired* set of sensory data generalize as well as those ANNs trained on all motor and tendon tension states when provided with at least 2 minutes of babbling data. Therefore, we conclude that it is possible to train an ANN on limited non-collocated measurements of *motor position, motor velocity, and tendon tension only* to reliably estimate joint angles during a variety of movements. Importantly, this novel bio-inspired state estimation framework with non-collocated sensors in tendon-driven systems can provide accurate joint angle estimation during dynamical tasks with limited data *if* tendon tension measurements are available.

While these results serve as an important proof-of-concept for such a framework, there are potential limitations based on the assumptions made. Specifically, these results were generated for a specific type of motor babbling, from networks of a specific design and set of hyper-parameters, and on particular mechanical plant. It is therefore critical that future work focus on quantifying the affects that changes to these parameters may have on the performance, learning, and robustness of networks designed to predict joint angles from limited-experience, non-collocated sensory data.

Lastly, this work has the important consequence of supporting the neurophysiological hypothesis that Golgi tendon organs (commonly assumed to be used to estimate joint torques) may in fact be a critical contributor to muscle spindle afferents to estimate joint kinematics. Such a co-evolutionary relationship has been conjectured in the past [28], and is now supported by our computational work.

SUPPLEMENTARY INFORMATION

The code used in this study and additional figures/movies can be accessed through the project's Github repository at: `https://github.com/danhagen/iO-IROS-2020`.

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