Anatomical Variability Naturally Leads to Multimodal Distributions of Denavit-Hartenberg Parameters for the Human Thumb

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Abstract—**A realistic biomechanical model of the thumb would enhance our understanding of the functional consequences of orthopedic and neurological diseases, and of treatments. Our work has shown that a kinematic description with five orthogonal and intersecting axes of rotation cannot predict realistic thumbtip forces. An alternative kinematic description proposes five axes of rotation that need not be orthogonal or intersecting. In order to make this description amenable for roboticist-use, we described the model in Denavit-Hartenberg (DH) notation. We explored the effects of reported anatomical variability on the DH parameters using Monte Carlo simulations. We report the DH parameters as statistical distributions that can be used for robotics-based models of the hand and stochastic analyses. We found three characteristic sets of kinematic descriptions. In 65.2% of the 3,140 simulations, the metacarpophalangeal flexion-extension axis was distal to the metacarpophalangeal adduction-abduction axis. We pose the question: Are multiple types of kinematic descriptions necessary to account for the natural anatomical variability of the thumb? This question is important for the biomechanical modeling of the hand, as the debate continues of whether patient-specific models are needed to simulate hand function for clinical applications, or if a single common model suffices.**

Keywords—**Biomechanical model, biorobotics, hand, kinematics, Monte Carlo simulation, thumb**

I. INTRODUCTION

From the most precise pinch to the most powerful grasp, the versatility and utility of the human thumb is evident whenever we use our hands to interact with objects. A realistic biomechanical model of the thumb would be instrumental to the study of the functional consequences of orthopedic and neurological diseases, and treatment outcomes.

Our modeling work has shown that assuming a kinematic description of the thumb with five orthogonal and intersecting axes of rotation at the carpometacarpal (CMC) and metacarpophalangeal (MP) joints cannot predict realistic thumbtip forces [1].

An alternative kinematic description proposes five axes of rotation that need not be orthogonal or intersecting [2]. The kinematic description of this virtual five-link model, however, is not in a format amenable for use in robotics-based models. Moreover, it is not known if the large variability in the anatomical data used to derive this description [3, 4] is informative of kinematic differences among individual thumbs. That is, the mean axis location and orientation values may not be representative of any one thumb.

The links of the model are "virtual" in that they correspond to the distance between consecutive effective hinges, and not simply the lengths of the three long bones of the thumb, as in previous models. Adjacent virtual links are connected to one another by one hinge. The CMC and MP joints each have two axes of rotation, one for flexion-extension (FE) and one for adduction-abduction (AA). The interphalangeal (IP) joint has one axis of rotation for flexion-extension. The virtual five-link model differs from its predecessors in that the FE and AA axes are not orthogonal to one another or to the long axes of the bones, and adjacent axes do not necessarily intersect one another within the bones of the thumb.

Denavit-Hartenberg (DH) notation is the standard approach in robotics to describe joint kinematics for computational applications. The computational versatility of the DH representation of robotic joints makes it the logical choice for describing the complex virtual five-link manipulator [5].

The objective of this work was twofold: to describe the virtual five-link model in Denavit-Hartenberg notation for use in robotics-based models of the hand; and to establish the effects of the reported anatomical variability on this kinematic description of the thumb.

II. METHODOLOGY

We calculated the four DH parameters (θ, d, a, and α describing the location and orientation of the axes of rotation) from the 2D projections of the axes reported by Hollister *et al.* [3, 4] by interpreting them as rotations and translations in 3D. According to Denavit-Hartenberg conventions, each axis of rotation was labeled a z-axis. The most proximal z-axis, the CMC FE axis, was labeled z_0. The labeling continued distally whereby the subscripts of the z-axes were incremented by one until the most distal axis of rotation, the IP FE or z_4 axis, had been reached.

We explored the effects of the reported anatomical variability of these axes [3, 4] using Monte Carlo (MC) simulations, one type of stochastic analysis technique [6]. At each of 3,140 MC iterations, a set of anatomical parameters was randomly selected from uniform distributions bounded

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by reported mean value ± one standard deviation [1, 3, 4]. This set of anatomical parameters was then transformed into a set of DH parameters.

Since a range of anatomical parameters was reported, a range of DH parameters resulted from the transformations. We considered convergence of the DH parameter distributions to have occurred once successive mean and coefficient of variance values changed by less than 1% for at least the last 20% of the simulations.

We characterized each DH parameter distribution using standard statistical distributions, such as the normal, beta, gamma, uniform, and normal mixture. Normal distribution parameters (μ,σ) were determined by the mean and standard deviation sample statistics. Beta distribution parameters (α,β) were estimated using maximum likelihood estimation. Gamma distribution parameters (α,β) were determined by non-parametric bootstrap sampling followed by the method of moments. Normal mixture parameters (π_i,μ_i,σ_i for i=1,2 or 1…3) were estimated using the expectation-maximization method [7].

III. Results

Multimodal distributions were observed in some DH parameters despite the initial uniform distributions of the anatomical parameters (Fig. 1). The MC simulations reveal that the reported anatomical variability results in three characteristic descriptions of thumb kinematics. In 65.2% of the simulations (sets 1 and 2), the MP FE axis was distal to the MP AA axis. In all others (set 3), this order was reversed. Sets 1 and 2 differed in that the MP FE axis was slightly dorsal to the IP FE axis in set 1 (29.2% of cases) and slightly palmar in set 2 (36.0% of cases). Fig. 2 depicts a sample case from set 1.

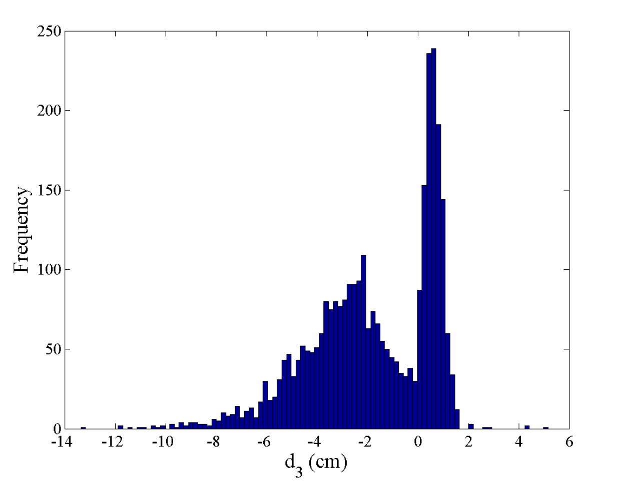

Fig. 1. Histogram of DH parameter d_3
This is one example of a bimodal distribution that resulted from the MC simulations. The distribution was characterized as a mixture of normal distributions, specified in the d_3 cell of Table 1.

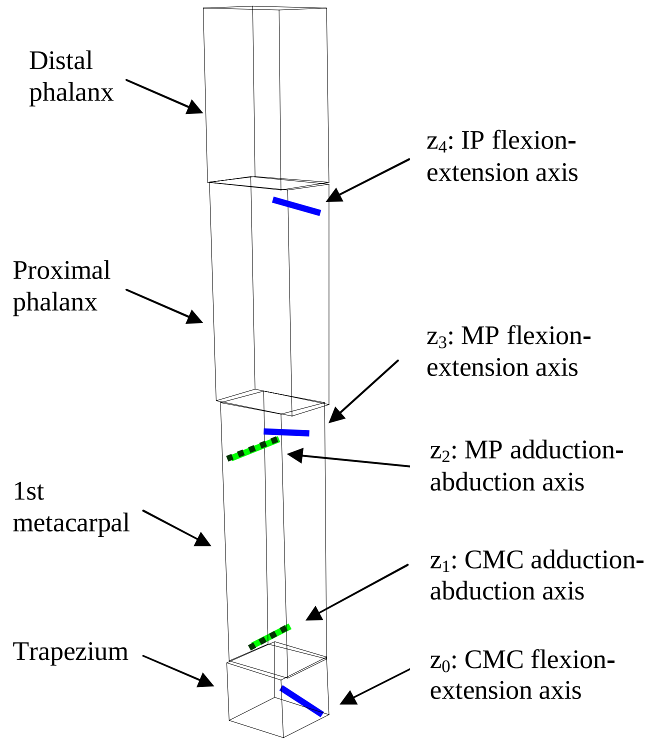

Fig. 2. Representative instantiation of the five axes of rotation, depicted by the bold lines, for set 1 (not to scale).

Table 1 contains the statistical characterization of each DH parameter. We present four sets of DH parameters for relating the location and orientation of consecutive axes of rotation for z_0 through z_5. The θ values are not specified because they are rotational degrees of freedom.

IV. Discussion

Monte Carlo simulations are affected by the number, distribution-type, range, and covariance of the variables to be randomly drawn. The large variability in the DH parameters (Table 1) may reflect the relatively large variability in the reported anatomical data [3, 4] and/or the fact that we did not set any parameter covariances. Not including these currently unknown covariances likely produced some unrealistic parameter combinations.

The inversion in the proximal/distal location of the MP FE and AA axes among iterations is not surprising given the overlapping distributions of reported MP axis parameters [4]. However, we had no reason to expect that there would be more cases in which the FE axis was distal to the AA axis. Understanding the kinematic differences between the two different cases of relative MP axis locations could be critical to the design of surgical techniques and the success of the clinical outcomes.

Since anthropometric data were not reported [3, 4], we do not know if factors such as subject sex, hand size, or

TABLE I
FOUR DH PARAMETER SETS SPECIFY RELATIVE LOCATIONS OF THE FIVE ROTATIONAL DEGREES OF FREEDOM. WE FOUND THREE CHARACTERISTIC SETS OF DH PARAMETERS. FOR PARAMETERS THAT DISTINGUISH THE SETS, THREE SETS OF STATISTICAL DISTRIBUTIONS ARE LISTED (SETS 1, 2, AND 3 IN ROMAN, *ITALICS*, AND **BOLD**, RESPECTIVELY). THE FOLLOWING NOTATION IS USED: NORMAL N (μ, σ), BETA B (α, β), GAMMA G (α, β), UNIFORM U (a, b), AND NORMAL MIXTURE π_i*N (μ_i, σ_i)

DH param.	Relationships of successive axes of rotation			
	$z_0 \rightarrow z_1$	$z_1 \rightarrow z_2$	$z_2 \rightarrow z_3$	$z_3 \rightarrow z_4$
a (cm)	N (1.22, 0.26)	N (3.23, 0.74)	B (0.85, 1.23)	B (1.30, 10.02) *B (1.30, 10.02)* **N (3.96, 0.39)**
d (cm)	N (-0.23, 0.25)	N (3.40, 2.16) *N (3.40, 2.16)* **0.46*N (0.11, 0.43), 0.54*N (1.33, 0.46)**	N (-3.11, 2.05) *N (-3.11, 2.05)* **0.72*N (0.49, 0.27), 0.28*N (0.97, 0.24)**	G (13.77, 1.06) *G (13.77, 1.06)* **0.74*N (-1.09, 0.33), 0.26*N (-0.44, 0.19)**
α (rad)	0.57*N (-1.50, 0.08); 0.43*N (-1.29, 0.07)	N (-0.58, 0.15) *N (-0.58, 0.15)* **0.29*N (1.20, 0.05), 0.71*N (1.41, 0.10)**	0.47*N (1.80, 0.04); 0.53*N (1.93, 0.05) *0.47*N (1.80, 0.04) 0.53*N (1.93, 0.05)* **0.53*N (-1.94, 0.05), 0.47*N (-1.80, 0.04)**	N (-0.31, 0.07) *N (0.31, 0.07)* **0.33*N (1.73, 0.03), 0.48*N (1.84, 0.05), 0.19*N (1.93, 0.02)**

anatomical variability contributed to the trends observed in the Monte Carlo simulations, or if the three characteristic sets of DH parameters are associated with these factors.

Future improvements to the kinematic description of the thumb involve the addition of translational degrees of freedom. Pearlman *et al*. [8] found that the trapezium, routinely assumed to be the fixed base of the thumb, translates under load. Thus, a truly realistic kinematic model of the thumb may need to include a trapezium with a load-dependent translational degree of freedom. Additional improvements include the use of contact theory to model the kinematics of arbitrarily-shaped articulating bone surfaces at the CMC joint, for example.

V. CONCLUSION

This work provides the Denavit-Hartenberg notation for a kinematic description of the thumb with five axes of rotation that need not be orthogonal or intersecting. The variability of the anatomical data used to derive this notation naturally resulted in multimodal statistical distributions of DH parameters. We are currently investigating the biomechanical consequences of the three sets of kinematic descriptions found, such as muscle coordination for thumb motion and force production.

The statistical distributions we report can serve as prior distributions for robotics-based Monte Carlo simulations of human manipulation. That is, pseudo-random sets of DH parameters can be drawn from these distributions obtained from the natural anatomical variability of the thumb [1, 3, 4]. Importantly, we must now pose the question: Are multiple types of kinematic descriptions necessary to faithfully account for the natural anatomical variability of the thumb? The answer to this question will guide the debate of whether patient-specific models are needed to simulate hand function for clinical applications, or if a single common model suffices.

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