

# VISUALIZING THE STRUCTURE OF HIGH DIMENSIONAL FEASIBLE ACTIVATION SETS FOR STATIC FORCE PRODUCTION

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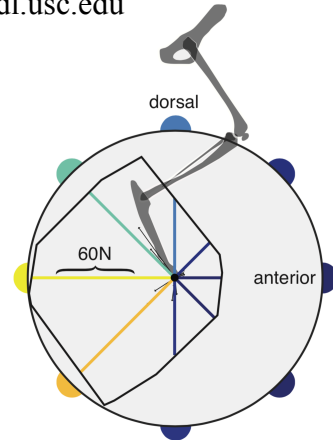
## INTRODUCTION

We present a novel ‘vectormap’ transformation to visualize the features of feasible force and feasible activation sets. Visualizing the polyhedra and high-dimensional polytopes corresponding to these feasible sets, respectively, is challenging. Prior work [1] used their bounding boxes, which overestimate their volume, and oversimplify their shape. Understanding the structure of these feasible sets is critical to inform the central debates in motor control such as muscle redundancy, synergies and motor learning.

## METHODS

The ‘vectormap’ is a radial projection of these polytopes onto the surface of their enclosing ball, where the resulting color-mapped sphere represents the size of the polytope in that direction (Fig. 1). As an example, we present the feasible force set of a feline hind limb, and its associated 31D feasible activation set. The parameters from the musculoskeletal model were shared with us by [2].

Extending Fig. 1 to higher dimensions, a similar idea applies. A vectormap sphere can display a muscle’s activation solution across all maximal forces, as shown on the right in Fig. 2. The production of a given sub-maximal force magnitude in a particular direction can be achieved by multiple coordination patterns [4,5]. Therefore, two vectormap spheres can be created for each muscle to provide a color map of the minimal and maximal activation for endpoint forces in all possible 3D directions.



**Figure 1:** 2D description of vectormap for the feasible force set of feline hind limb in the sagittal plane. The polygonal feasible force set represents the maximal static force in each radial direction. That maximal force value is encoded as the color of the surface of the enclosing circle in that same direction. This same approach is extended to higher dimensions.

## RESULTS AND DISCUSSION

Figure 1 shows a 2D slice of a vectormap for the feasible force set, and Figure 2 shows the activation at different levels of force magnitude for *vastus lateralis* and *medial gastrocnemius* as sample muscles. [http://bbdl.usc.edu/vector\\_mapping/](http://bbdl.usc.edu/vector_mapping/) shows the vectormaps of feasible activation for all 31 muscles. The left side of Fig. 2 shows those two vectormap spheres for different levels of force output, revealing how redundancy of activation is lost for each muscle as we march towards the maximal feasible force. These plots provide a detailed view of the structure of the 31D feasible activation set, one muscle at a time. For example, note that the lower and upper bounds naturally converge for maximal output, but they converge at different rates across muscles (vectormap progressions for all 31 muscles are not illustrated here) and directions of force output.

Interestingly, we find that the upper and lower bounds are more sensitive to than expected, with the bounds changing dramatically between vectors less than a degree apart in direction. We find that activation vectormaps can show strong discontinuities depending on the direction of output force, even at low magnitudes. Common tasks like locomotion require changes in force direction, thus muscle redundancy, synergies, and motor learning must be considered from a spatiotemporal perspective [3]. The vectormap approach allows us to pose and test musculoskeletal hypotheses for generalized force outputs, even for high-dimensional musculoskeletal systems.

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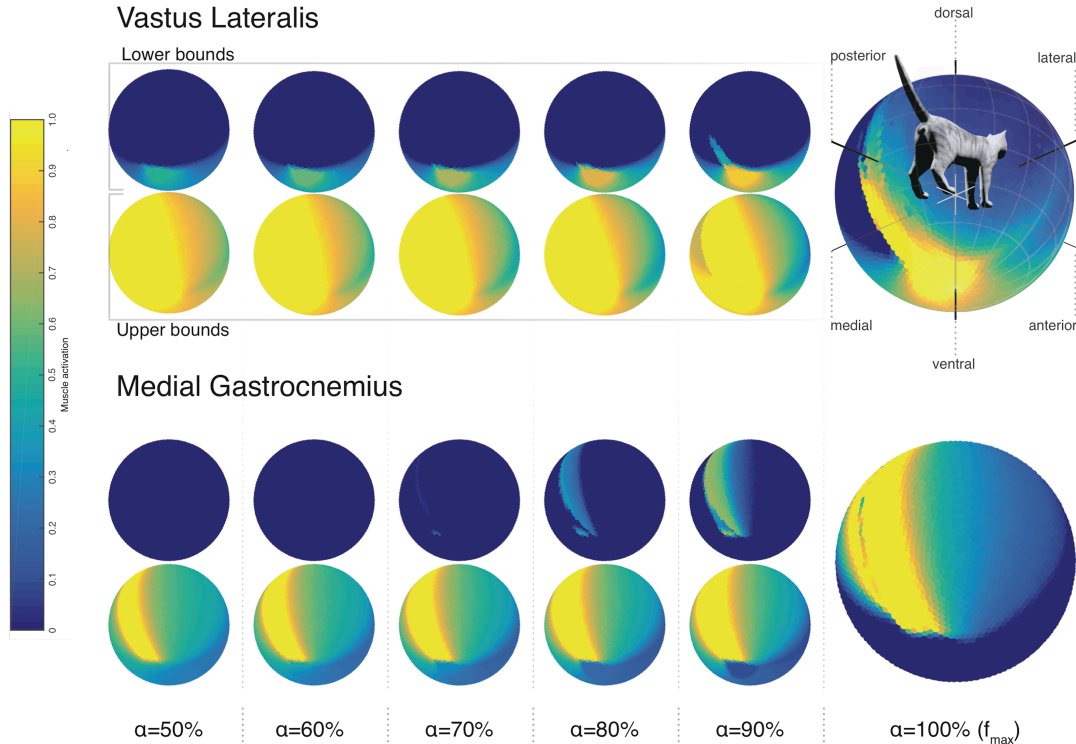
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**Figure 2:** Vectormap visualization of one cat's feasible activation space for *vastus lateralis* and *medial gastrocnemius*. Activation lower (top) and upper bounds (bottom) are shown for 50%, 60%, 70%, 80%, 90% and 100% of maximal force (from left to right). Vectormap visualizations exist for the other 29 muscles in the same format. A muscle activation of 1.0 is 100% MIC.