Incorporating Muscle Activation-Contraction dynamics to an optimal control framework for finger movements

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Introduction

Recent experimental and theoretical work [1] investigated the neural control of contact transition from motion to force during tapping with the index finger as a non-linear optimization problem. Such transitions from motion to well-directed contact force are a fundamental part of dexterous manipulation.

Experimental work in [2] shows that the nervous system selects an initial change in direction with a subsequent change in magnitude of the torque vector. In [1] we suggest that this may in fact be an optimal strategy. In [4] the framework of Iterative Linear Quadratic Optimal Regulator (ILQR) was extended to incorporate motion and force control. However, our prior simulation work assumed direct and instantaneous control of joint torques, which ignores the known delays and filtering properties of skeletal muscles.

In this first study of how ILQR behaves with respect to neuromuscular delays:

1) We created a torque driven a 3 link-planar model of the finger that includes delays associated with activation contraction dynamics of muscle.

2) We apply ILQR to this more plausible nonlinear model of the finger to find the latent control commands, u, to recreate an experimental tapping movement prior to contact.

3) As a first step to study the controllability of the system, we investigated the controllability of the linear mapping from the latent control signal to the joint torques.

Methods: Iterative Linear Quadratic Regulator Control (ILQR)

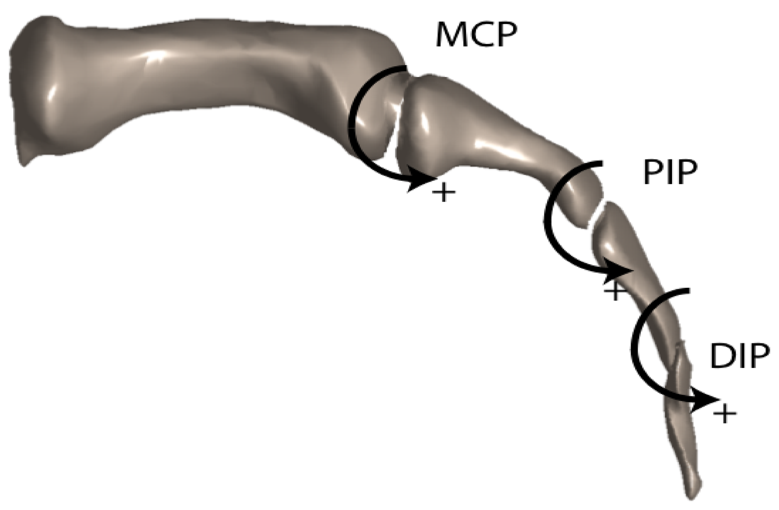
In the optimal control framework the goal is to minimize the objective function below subject to the constraints imposed by the dynamics of the system under consideration.

$$V = \frac{1}{2}\left(x(t_f) - x^*\right)^T Q_f \left(x(t_f) - x^*\right) + \frac{1}{2}\int_0^{tf}\left(x^T Q x + u^T R u\right) dt \quad (1)$$

$$\dot{x} = F(x,\tau) \quad (2)$$

The assumptions for the application of the Linear Optimal Control framework are
1) Quadratic or locally quadratic cost function with respect to the state and controls.
2) Q and R matrices in the cost function have to be positive definite - convexity.
3) Observable plant dynamics - Detectable cost function.
4) Controllable plant dynamics

ILQR:
1) It does not require an initial control policy.
2) It can be applied to Nonlinear Systems.



The inverse dynamics of the model finger are specified by the following equations

$$M(\theta)\ddot{\theta} + C(\theta,\dot{\theta}) = \tau \quad (3)$$

$$x = \left[\theta_{MCP}\ \theta_{PIP}\ \theta_{DIP}\ \dot{\theta}_{MCP}\ \dot{\theta}_{PIP}\ \dot{\theta}_{DIP}\right] \quad (4) \qquad \tau = \left[\tau_{MCP}\ \tau_{PIP}\ \tau_{DIP}\right] \quad (5)$$

For the case of ILQR the objective function under minimization is expressed as follows:

$$V = \frac{1}{2}\left(x_N + \delta x_N - x^*\right)^T Q_f \left(x_N + \delta x_N - x^*\right) + \frac{1}{2}\sum_{k=0}^{N-1}\left\{\left(x_k + \delta x_k\right)^T Q\left(x_k + \delta x_k\right) + \left(u_k + \delta u_k\right)^T R\left(u_k + \delta u_k\right)\right\} \quad (6)$$

Subject to the linearized dynamical constraints:

$$\delta x_{k+1} = A_k \delta x_k + B_k \delta u_k \quad (7)$$

Methods, contd.

The state and control transition matrices for the linearized model are given as follows:

$$A_k = I + \left.\frac{\partial F(x,u)}{\partial x}\right|_{x_k} \quad (8) \qquad B_k = \left.\frac{\partial F(x,u)}{\partial u}\right|_{u_k} \quad (9)$$

The ILQR controller consist of the equation that follow:

$$\delta u_k = -K\delta x_k - K_v v_{k+1} - K_u u_k$$
$$K = \left(B_k^T S_{k+1} B_k + R\right)^{-1} B_k^T S_{k+1} A_k$$
$$K_v = \left(B_k^T S_{k+1} B_k + R\right)^{-1} B_K^T,$$
$$S_k = A_k^T S_{k+1}\left(A_k - B_k K\right) + Q,$$
$$v_k = \left(A_k - B_k K\right)^T v_{k+1} - K^T R u_k + Q x_k \quad (10)$$

Terminal Conditions:

$$S_N = Q_f \qquad v_N = Q_f\left(x_N - x^*\right)$$

Torque delays:

$$\dot{\tau} = \Phi\tau + \Gamma u \quad (11)$$

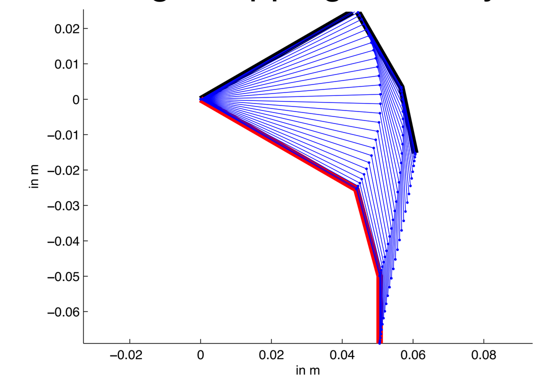
Kalman Controllability Test:

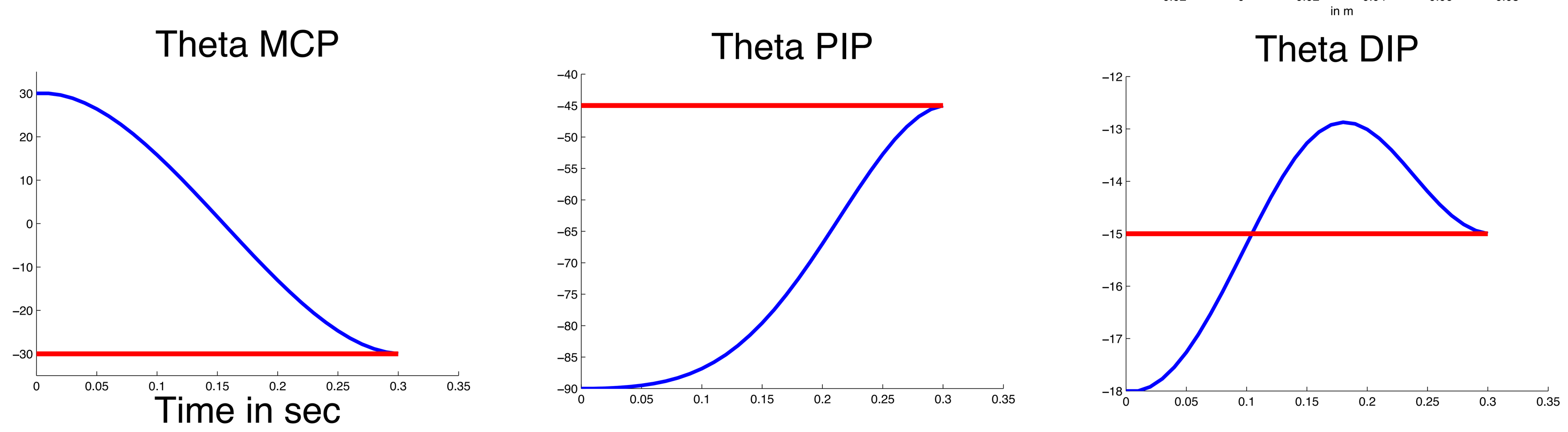
$$Rank\left[\Phi^{n-1}\Gamma \mid ... \mid \Phi\Gamma \mid \Gamma\right] \quad (12)$$

Simulations and results

Simulation1, sanity-check: Bring the finger from the initial posture (30°, -90°, -18°) to target configuration (-30°, -45°, -15°) with zero terminal velocity during the time horizon of 300ms.

Finger tapping no delays

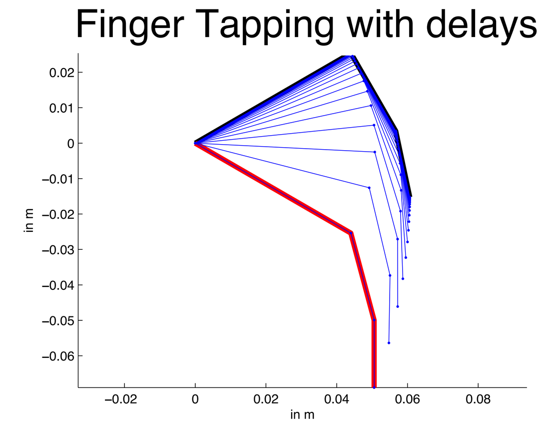


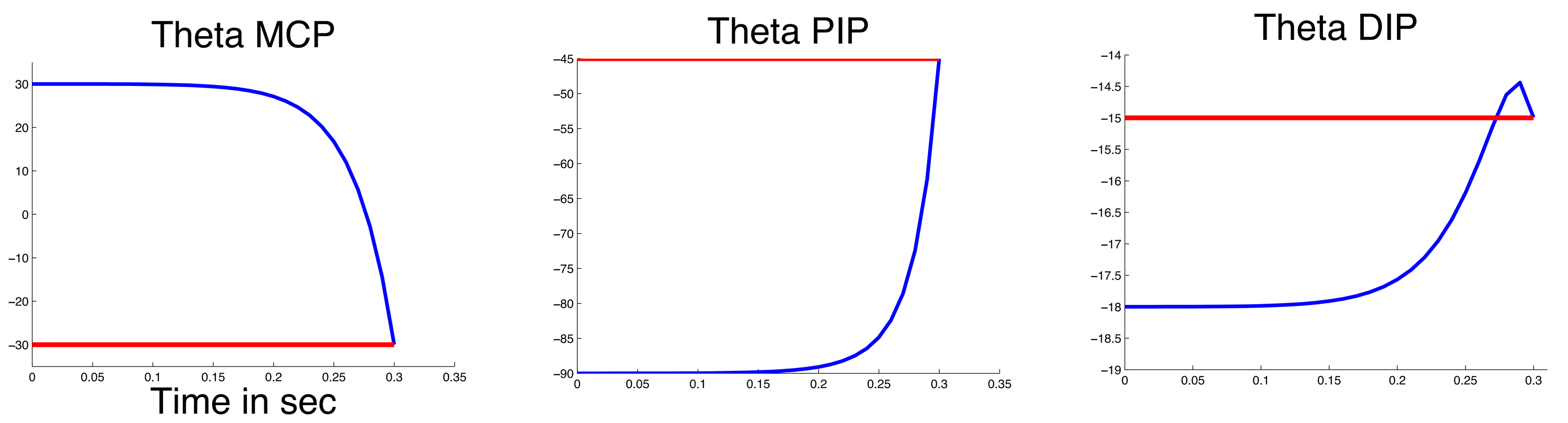


Simulation 2, using first order delays: The singular values of the controllability Grammian are a measure of sensitivity of the state with respect to controls. With high singular values(>10),(35,35,35), small changes in the controls lead to large changes in the controllable state space.

Kalman Controllability Test: $SVD\left[\Phi^{n-1}\Gamma \mid ... \mid \Phi\Gamma \mid \Gamma\right]$

Finger Tapping with delays





Conclusions

1) The need for fast actuation for even simple tasks may have influenced the evolution, the type and the number of muscles and their tendon routing. We will continue to explore the interaction between the necessary and sufficient control requirements and neuro-muscular structure.

2) In future work, we will aslo apply ILQR to a tendon driven model which contains 7 tendons that include passive structure such as skin. In addition, the controllability characterization of the finger and the hand model will be investigated.

REFERENCES
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