The Boundary of Instability as a Powerful Experimental Paradigm for Understanding Complex Dynamical Sensorimotor Behavior: Dexterous Manipulation as an Example

Madhusudhan Venkadesan[1], Francisco J. Valero-Cuevas[1] and John M. Guckenheimer[2]

[1] Department of Mechanical & Aerospace Engineering, Cornell University, Ithaca, NY, U.S.A.
[2]Department of Mathematics, Cornell University, Ithaca, NY, U.S.A.
E-mail: mv72@cornell.edu Web: http://www.mae.cornell.edu/nmbl

INTRODUCTION

Dynamical motor behavior is often extremely complex and nonlinear. Bifurcation theory (from nonlinear dynamics) predicts that even complex nonlinear systems, like the brain-hand sensorimotor function during manipulation, behave like low-order systems, when at the verge of instability [1]. Moreover, activation dependent noise in fingertip force and position influences the dynamical behavior of the brain-hand-object system. Thus, we propose a stochastic dynamical system model to characterize the complex dynamical behavior of the brain-hand-object system on the verge of instability. In this study, we use a novel experimental paradigm by bringing the system to the verge of instability, which we model as a stochastic pitchfork bifurcation. We show remarkable similarity of the simulated results to observed data. This justifies the use of bifurcation analysis to understand sensorimotor function in dynamic manipulation, and clinically grade its impairment.

EXPERIMENTAL METHODS

As in our previous work [4], 10 consenting unimpaired subjects (29±7 years, 5 males) used their thumbtip to compress a slender spring prone to buckling (Fig 1). The instructions to the subjects were to "*compress the spring as far as you can, even if the spring oscillates or is not straight, and hold the spring at this shortest length for a few seconds*." We collected 3D acceleration, compressive spring force in all subjects, and 3D thumbtip position/orientation in 5 subjects (12-camera system). We measured static maximal key and opposition pinch strength. The calculated buckling loads [2] for the first 3 modes of this spring are: 1.6N, 7.0N and 15.2N.

RESULTS AND DISCUSSION

Performance as measured by maximal compressive spring force (23.8 N ± 11%) is only slightly dependent (Fig 2) on their pinch strength (Key: 87.7 N ± 24%, Opposition:70.7 N ± 29%).

A clear onset of instability (bifurcation) was observed in 3-D position data past a critical load (Fig 3). Moreover, the projection of the position data close to the instability onto a horizontal plane (Fig 3) lies along a straight line ($r^2 \geq 0.8$). This indicates that 1-D dynamics is the dominating behavior. The projection of the data before the onset of instability does not show any such 1-D behavior (Fig 3). So, the dimension reduction is a consequence of the dynamics near buckling and not just the thumb's mechanics.

Milstein's method [3] was used to integrate the equation for the stochastic pitchfork bifurcation:

$$dx = (\alpha(\mu - \mu_0)x - \beta x^3 + \varepsilon)dt + \sigma \cdot dW \qquad (1)$$

α and β are scaling parameters, μ, μ_0 are the current and buckling loads, ε is the imperfection in the system, σ is the overall noise level and dW is a unit Brownian process. A comparison of the simulation to collected data (Fig 4) indicates that the simulation results are qualitatively similar to the observed data.

We conclude that the maximal compression of the spring is consistent across people of different strengths. Moreover, the behavior of the brain-hand-spring system at the verge of instability seems to be qualitatively well characterized by a stochastic pitchfork bifurcation. Based on this preliminary work, we propose the boundary of instability as a powerful experimental paradigm for understanding complex dynamical sensorimotor behavior. We will use this method to quantitatively grade clinically relevant impairment of manipulation ability in future.

REFERENCES

1. Guckenheimer, J. and Holmes, P. (1983) *Nonlinear oscillations, dynamical systems and bifurcations of vector fields*. Springer-Verlag.
2. Haringx, J.A. (1948) *Philips Research Reports* **3**(6), 401-449.
3. Higham, D.J. (2001). *SIAM Review* **43**(3), 525-546.
4. Valero-Cuevas, F.J., et al. (2003). *J* Biomech **36** , 265-270.

ACKNOWLEDGEMENTS

Prof. Jessica Hodgins and Jin-Xiang Chai at CMU for the motion capture system. Also, we gratefully acknowledge support from the NSF CAREER award BES-0237258 to FVC.

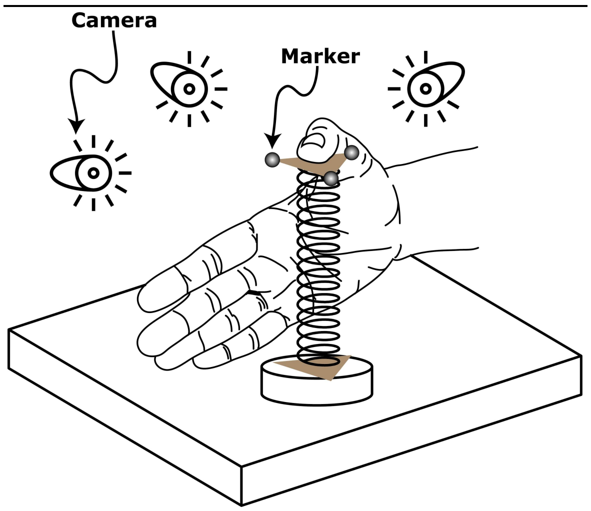

Figure 1: A schematic diagram to show the experimental setup used. The unused digits and arm of the subject were restrained using a flat, vertical plate in the actual experiment.

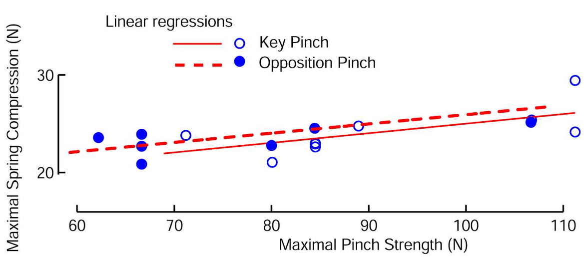

Figure 2: The plot above shows that maximal spring compression is only slightly dependent on strength since the slopes of both the regression lines are close to 0 and r^2 is not high (0.61and 0.62 for key and opposition respectively).

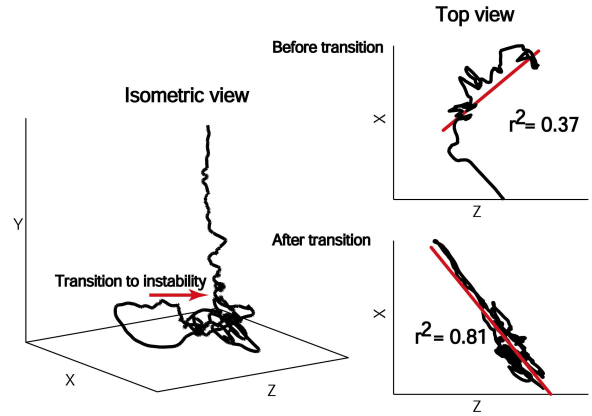

Figure 3: A typical sample 3D data that shows a clear onset of instability. Also, it demonstrates that the dynamics are limited to 1-D past the critical point, but there is no such dimension reduction of the behavior prior to the onset of instability.

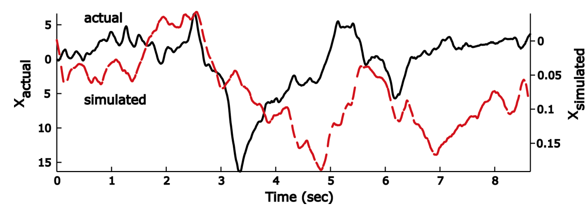

Figure 4: The solid line represents a sample data from one subject, while the dotted line represents the result of numerically simulating the normal form of a stochastic pitchfork bifurcation. Qualitatively, the simulation is strikingly similar to the experimental data. Note that the simulation is on a normalized spatial scale.